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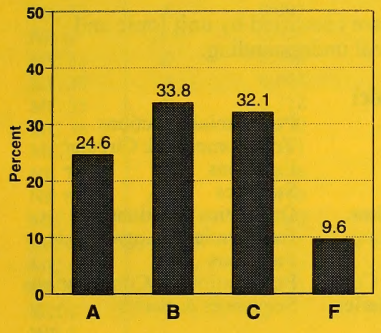
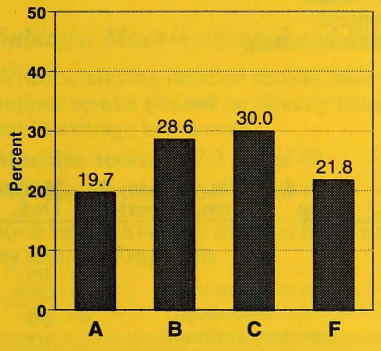
# Mathematics 30

## Diploma Examination Results
## Examiners' Report for June 1998

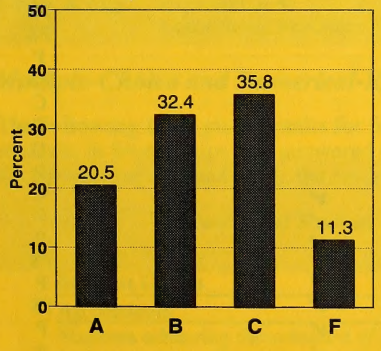
**School-Awarded Mark**




**Diploma Examination Mark**




**Final Course Mark**




The summary information in this report provides teachers, school administrators, and students with an overview of results from the June 1998 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used in conjunction with the detailed school and jurisdiction reports that are provided electronically to schools and school jurisdiction offices. A provincial report containing a detailed analysis of the combined November, January, June, and August results is made available annually.

### *Description of the Examination*

The Mathematics 30 Diploma Examination consists of 40 multiple-choice questions worth 57.1%, 9 numerical-response questions worth 12.9%, and 3 written-response questions worth 30% of the total examination mark.

### *Achievement of Standards*

The information reported is based on the final course marks achieved by 8 432 students who wrote the June 1998 examination.

- 88.7% of the 8 432 students achieved the acceptable standard (a final course mark of 50% or higher).
- 20.5% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 50.6% of the students who wrote the June 1998 examination were female.

- 88.5% of the female students achieved the acceptable standard (a final course mark of 50% or higher).
- 18.6% of these female students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 49.4% of the students who wrote the June 1998 examination were male.

- 88.8% of the male students achieved the acceptable standard (a final course mark of 50% or higher).
- 22.3% of these male students achieved the standard of excellence (a final course mark of 80% or higher).

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## *Provincial Averages*

- The average school-awarded mark was 67.6%.
- The average diploma examination mark was 63.5%.
- The average final course mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 65.9%.

Of the 8 432 students who wrote the June 1998 examination, 16% had written at least one Math 30 exam previously.

## *Results and Examiners' Comments*

This examination has a balance of question types and difficulties. It is designed so that students capable of achieving the acceptable standard will obtain a minimum mark of 50%, and students capable of achieving the standard of excellence will obtain a minimum mark of 80%.

In the following table, diploma examination questions are classified by question type: multiple choice (MC), numerical response (NR), and written response (WR). The column labelled "Key" indicates the correct response for multiple-choice and numerical-response questions. For numerical-response questions, a limited range of answers was accepted as being equivalent to the correct answer.

For multiple-choice and numerical-response questions, the "Difficulty" indicates the proportion (out of 1) of students answering the question correctly.

Questions are classified by unit topic and mathematical understanding.

**Unit Topic:**

| | |
|---|---|
| Poly. Fn. | Polynomial Functions |
| Trig. Fn. | Trigonometric & Circular Functions |
| Stat. | Statistics |
| Quad. Rltns. | Quadratics Relations |
| Exp. & Log. | Exponential & Logarithmic Functions |
| Perm. & Com. | Permutations & Combinations |
| Seq. & Series | Sequences & Series |

**Mathematical Understandings:**

| | |
|---|---|
| P | Procedure |
| C | Concept |
| PS | Problem-solving |

### *Blueprint*

| Question | Key | Difficulty | Poly. Fn. | Trig. Fn. | Stat. | Quad. Rltns. | Exp. & Log. | Perm. & Com. | Seq. & Series | Math Und. |
|---|---|---|---|---|---|---|---|---|---|---|
| MC 1 | D | 0.763 | ✔ | | | | | | | C |
| MC 2 | C | 0.740 | ✔ | | | | | | | PS |
| MC 3 | C | 0.473 | ✔ | | | | | | | PS |
| MC 4 | B | 0.722 | ✔ | | | | | | | C |
| MC 5 | B | 0.539 | ✔ | | | | | | | PS |
| MC 6 | D | 0.499 | ✔ | | | | | | | PS |
| MC 7 | A | 0.787 | ✔ | | | | | | | C |
| NR 1 | 2314 | 0.873 | ✔ | | | | | | | C |
| MC 8 | C | 0.670 | | ✔ | | | | | | PS |
| MC 9 | C | 0.522 | | ✔ | | | | | | PS |
| MC 10 | C | 0.447 | | ✔ | | | | | | P |
| MC 11 | D | 0.459 | | ✔ | | | | | | PS |
| MC 12 | B | 0.844 | | ✔ | | | | | | C |
| MC 13 | A | 0.745 | | ✔ | | | | | | C |
| NR 2 | 0.95 | 0.477 | | ✔ | | | | | | C |
| NR 3 | 1243 | 0.435 | | ✔ | | | | | | P |
| MC 14 | C | 0.878 | | | | | ✔ | | | C |
| MC 15 | C | 0.407 | | | | | ✔ | | | C |
| MC 16 | A | 0.820 | | | | | ✔ | | | P |
| MC 17 | D | 0.592 | | | | | ✔ | | | PS |
| MC 18 | D | 0.777 | | | | | ✔ | | | P |
| MC 19 | B | 0.814 | | | | | ✔ | | | PS |
| MC 20 | B | 0.653 | | | | | ✔ | | | P |
| NR 4 | 729 | 0.681 | | | | | ✔ | | | P |
| MC 21 | A | 0.862 | | | | ✔ | | | | C |
| MC 22 | B | 0.743 | | | | ✔ | | | | C |
| MC 23 | B | 0.409 | | | | ✔ | | | | PS |

| Question | Key | Difficulty | Poly. Fn. | Trig. Fn. | Stat. | Quad. Rltns. | Exp. & Log. | Perm. & Com. | Seq. & Series | Math Und. |
|---|---|---|---|---|---|---|---|---|---|---|
| MC 24 | D | 0.771 | | | | ✔ | | | | C |
| MC 25 | A | 0.832 | | | | ✔ | | | | C |
| MC 26 | C | 0.646 | | | | ✔ | | | | C |
| NR 5 | 17.0 | 0.300 | | | | ✔ | | | | P |
| MC 27 | B | 0.784 | | | | | | | ✔ | P |
| MC 28 | C | 0.468 | | | | | | | ✔ | PS |
| MC 29 | D | 0.531 | | | | | | | ✔ | PS |
| MC 30 | A | 0.721 | | | | | | | ✔ | PS |
| MC 31 | A | 0.651 | | | | | | | ✔ | C |
| MC 32 | A | 0.597 | | | | | | | ✔ | PS |
| NR 6 | 75.0 | 0.628 | | | | | | | ✔ | P |
| MC 33 | D | 0.786 | | | | | | ✔ | | C |
| MC 34 | A | 0.862 | | | | | | ✔ | | C |
| MC 35 | C | 0.712 | | | | | | ✔ | | P |
| MC 36 | B | 0.730 | | | | | | ✔ | | P |
| MC 37 | A | 0.591 | | | | | | ✔ | | C |
| NR 7 | 720 | 0.846 | | | | | | ✔ | | P |
| NR 8 | 6720 | 0.510 | | | | | | ✔ | | P |
| MC 38 | D | 0.843 | | | ✔ | | | | | C |
| MC 39 | B | 0.671 | | | ✔ | | | | | P |
| MC 40 | A | 0.734 | | | ✔ | | | | | PS |
| NR 9 | 0.02 | 0.493 | | | ✔ | | | | | P |
| WR 1 | - | | | | | | | | | P/C/PS |
| WR 2 | - | | | | | | | | | P/C/PS |
| WR 3 | - | | | | | | | | | P/C/PS |

## *Subtests: Machine Scored and Written Response (Average by Subtest)*

When analyzing detailed results, bear in mind that subtest results **cannot** be directly compared. Results are in average raw scores.

**Machine scored:** 32.2 out of 49

**Written response:** 8.7 out of 15

### Raw Score Average for Machine-Scored Questions by Course Emphasis

| | | |
|---|---|---|
| Poly. Fn | Polynomial Functions | 5.3 out of 8 |
| Trig. Fn | Trigonometric & Circular Functions | 4.6 out of 8 |
| Stat | Statistics | 2.7 out of 4 |
| Quad. Rltns | Quadratic Relations | 4.6 out of 7 |
| Exp. & Log. | Exponential & Logarithmic Functions | 5.6 out of 8 |
| Perm. & Com. | Permutations and Combinations | 5.0 out of 7 |
| Seq. & Series | Sequences and Series | 4.4 out of 7 |

### Raw Score Average for Machine-Scored Questions by Mathematical Understandings*

- Procedural (P): 9.5 out of 15
- Conceptual (C): 14.0 out of 19
- Problem Solving (PS): 8.7 out of 15

* Refer to Appendix C of the 1997–98 *Mathematics 30 Information Bulletin, Diploma Examinations Program* for an explanation of mathematical understandings.

## *Multiple-Choice and Numerical-Response Questions*

The following table gives results for four questions selected from the examination and shows the percentage of students in four groups that answered the question correctly. The comments following the table discuss some of the understandings and skills the students may have used to answer these questions.

***Percentage of Students Correctly Answering Selected Machine-Scored Questions***

| Student Group | Question Number NR 3 | MC 9 | MC 23 | MC 24 |
|---|---|---|---|---|
| All Students | 43.5 | 52.2 | 40.9 | 77.1 |
| Students achieving the *standard of excellence* (80% or higher, or A) on the whole examination | 80.8 | 85.7 | 73.4 | 87.9 |
| Students achieving the *acceptable standard* (between 50% and 79%, B or C) on the whole examination | 41.7 | 51.7 | 36.8 | 76.6 |
| Students who have not achieved the *acceptable standard* (49% or less, or F) on the whole examination | 14.5 | 23.1 | 22.6 | 68.4 |

**Numerical Response**

**3.** Evaluate each of the following four trigonometric expressions. Identify the order of the values from smallest to largest. Record this order by listing the number of each expression, starting in the extreme left column of numerical response 3.

**1** $\tan^2\theta - \sec^2\theta$

**2** $\tan\theta - \dfrac{\sin\theta}{\cos\theta}$

**3** $5\sin^2\theta + 5\cos^2\theta$

**4.** $\dfrac{1}{7}\sin\theta \csc\theta$

(Record your answer in the numerical-response section of the answer sheet.)

**Answer: 1243**

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**9.** The amplitude of a given sine function, $f(\theta) = a \sin \theta + d$, is 6. If the maximum value of $f$ is 8, then the minimum value of $f$ is

**A.** –8
**B.** –6
***C.** –4
**D.** 2

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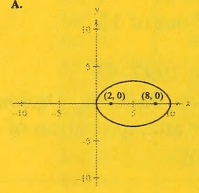
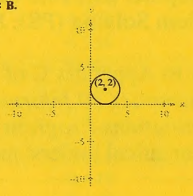
**23.** A quadratic relation is given by the equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where $A \times C \times D \times E = 0$. The foci of the ellipse and hyperbola, the centre of the circle, and the vertex of the parabola are shown. Which of the following graphs could **not** be the graph of this quadratic relation?

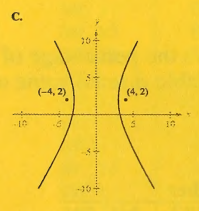
**A.**



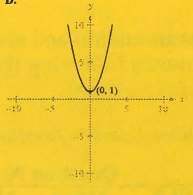

***B.**




**C.**




**D.**




The multiple-choice and numerical-response sections of the examination comprise questions that represent all content areas in Mathematics 30. A discussion of students' achievement of the curriculum standards in the units Trigonometric and Circular Functions, and Quadratic Relations follows.

**Trigonometric and Circular Functions** — To achieve the acceptable standard in trigonometric and circular functions, the students must be able to convert angle measurements between degree and radian measure, and given any two of the radian measure of the central angle, the radius, and the length of an arc, determine the unknown measurement. Students must also be able to verify the fundamental trigonometric identities; solve first-degree trigonometric equations on the domain $0 \le \theta < 2\pi$ in radians and $0° \le \theta < 360°$; and simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities.

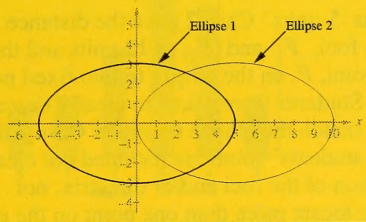
In addition, students must be able to generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages; explain the effect of each parameter $a$, $b$, $c$, and $d$ on the graph of the $y = a \sin [b(\theta + c)] + d$ and $y = a \cos [b(\theta + c)] + d$ functions; and be able to state the domain and range of $y = \sin \theta$, $y = \cos \theta$, and $y = \tan \theta$. Multiple-choice questions 8, 9, 10, 12, and 13, and numerical-response questions 2 and 3 require students to demonstrate their understanding of this unit.

In addition to the expectations for the acceptable standard, students who achieve the standard of excellence must be able to prove trigonometric identities and solve second-degree trigonometric equations, including double and half angles on the domains $0 \le \theta < 2\pi$ and $0° \le \theta < 360°$. They must be able to explain both orally and in writing the combined effects of the parameters $a$, $b$, $c$, and $d$ on the graphs of $y = a \sin [b(\theta + c)] + d$ and $y = a \cos [b(\theta + c)] + d$ and on the functions domain and range. Multiple-choice question 11 requires this of students.

**Quadratic Relations** —To achieve the acceptable standard in quadratic relations, students must be able to describe orally, in writing, and by modelling, each of the following: the intersection of a plane and a conical surface that would result in a hyperbola, an ellipse, a parabola, and a circle. They must also be able to identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate ellipse and hyperbola. Students must be able to describe orally and in writing each of the following: the quadratic relation defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$; the quadratic relation formed when given the value of the eccentricity; the eccentricity when given the quadratic relation; the quadratic

*Use the following information to answer the next question*

Two ellipses with identical shapes are shown below. The graph of ellipse 2 is obtained by moving the graph of ellipse 1 five units to the right.




**24.** Which of the following statements about the ellipses is **true**?

**A.** The values of the eccentricities of the ellipses are different.

**B.** The general equation of each ellipse is $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where the values for parameters $A$, $C$, and $F$ are the same.

**C.** The foci are closer together in the first ellipse than in the second.

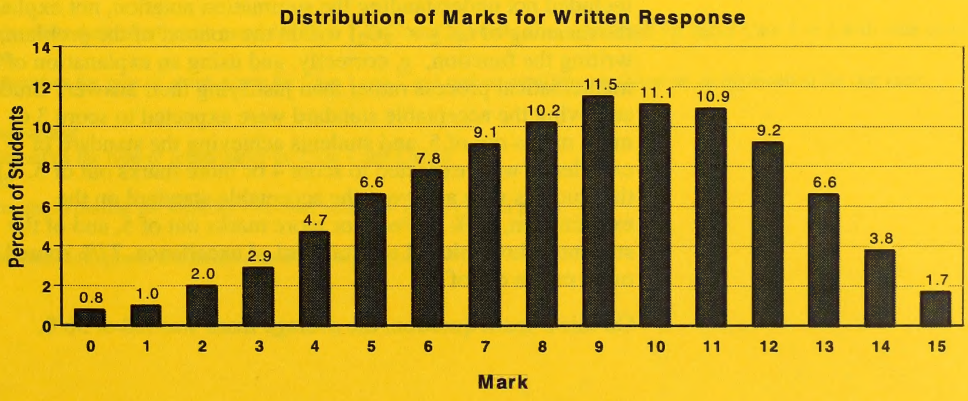
***D.** The sum of the distances from a point on an ellipse to its foci is the same for each ellipse.

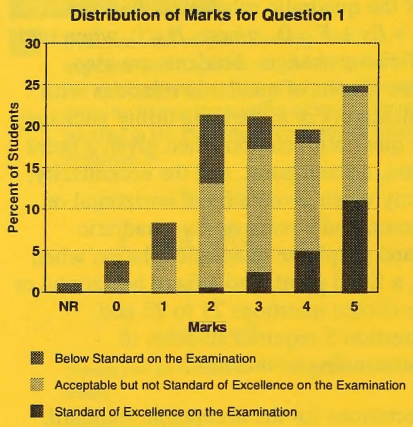
relation formed when given the locus definition; and the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two of the numerical coefficients change. Students are also expected to generate the graphs of quadratic relations with the use of graphing calculators or a graphing utility package; identify and graph the quadratic relation when given a point on the quadratic relation, a fixed point, and the eccentricity; calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a point on the quadratic relation; and identify and graph the quadratic relation when given the eccentricity, a fixed point, and a fixed horizontal or vertical line. Multiple-choice questions 21 to 26 and numerical-response question 5 required students to demonstrate their understanding of this unit.

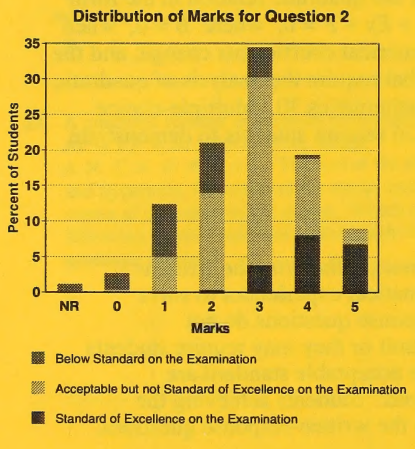
In addition to the expectations for the acceptable standard, students who achieve the standard of excellence must also be able to identify and to describe orally, in writing, and by modelling, the position of the plane at which the intersection of a plane and a conical surface defines a degenerate parabola; the changes in the graph of a quadratic relation when the eccentricity changes; the locus definition and use it to verify the equation of each conic section; the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change; and the solution to problems that require the analysis of quadratic relations studied in Mathematics 30. Multiple-choice questions 21, 24, and 26 require students to demonstrate these objectives.

## *Written-Response Questions*

As published in the 1998–99 *Mathematics 30 Diploma Examination Information Bulletin*, the written-response questions assess whether or not students can draw on their mathematical experiences to solve problems and to explain mathematical concepts. Therefore, the written-response questions do not necessarily fall into a particular unit of study but may cross more than one unit or they may require students to make connections among mathematical concepts. Students achieving the acceptable standard are expected to obtain at least 8 out of 15 marks on the written-response questions. Students achieving the standard of excellence are expected to obtain at least 12 out of 15 marks on the written-response questions.





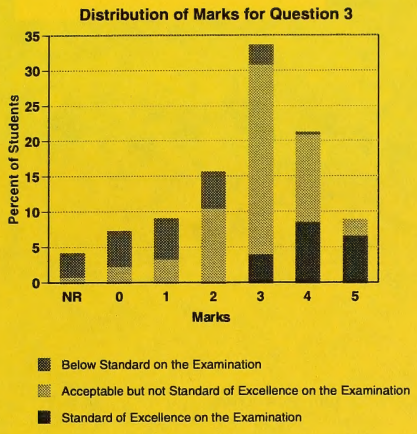

**Question 1** required students to identify which of three types of graph paper would be most appropriate on which to draw ellipses for two distinct cases. Case 1 provided the value of the eccentricity of an ellipse as $\frac{2}{3}$ and the distance between the directrix and the focus as 5 units. Case 2 gave the distance between an ellipse's two foci, $F_1$ and $F_2$, as 12 units and the sum of the distances from a point, $P$, on the ellipse to two fixed points, $F_1$ and $F_2$, as 14 units. Students were asked to clearly describe the steps they would take to sketch the ellipse for either Case 1 or Case 2. Examples of flaws in students' solutions included not clearly communicating the location of the foci and/or directrix, not describing clearly how to locate more than one point on the ellipse, and not explaining that the points had to be joined to form an ellipse.

Of the students who met the acceptable standard of achievement on the examination, 76% received at least 3 out of 5 marks, and of students who achieved the standard of excellence on the examination, 83% scored 4 or more marks out of 5.

On this 5-mark question, the average mark was 3.2 or 64%.

**Question 2** provided students with a function $f(n) = 2^n$, $n \in N$, which represented the number of ways that you can place $n$ distinguishable balls in two jars of different sizes. A diagram showed ways of placing a black ball and a white ball in two different sized jars. Students were asked to complete a chart so that it showed the relation between any number of distinguishable balls from 1 through 5 inclusive, and the number of ways the balls could be placed in two jars of different sizes. Students were asked to evaluate $\sum_{k=1}^{12} f(k)$ and, given an expression involving combinations that also represented the number of ways four balls of different colours could be put into two different jars, to explain the meaning of one of the terms from the expression $({}_4C_2 \times {}_2C_2)$ in the context of the problem. Finally, students were required to determine a function, $g$, that would represent the number of ways $n$ distinguishable balls can be placed in two identical jars and justify their answer. Examples of common flaws in students' solutions included not understanding the summation notation, not explaining the meaning of $({}_4C_2 \times {}_2C_2)$ within the context of the problem, not writing the function, $g$, correctly, and using an explanation of a mathematical process rather than justifying their answer. Students achieving the acceptable standard were expected to score 3 or more marks out of 5, and students achieving the standard of excellence were expected to score 4 or more marks out of 5. Of the students who achieved the acceptable standard on the examination, 75% scored 3 or more marks out of 5, and of the students who achieved the standard of excellence, 77% scored 4 or more marks out of 5.

On this 5-mark question, the average mark was 2.8 or 56%.




**Question 3** provided students with two correctly completed tables involving a geometric sequence $t_n$. Using the information provided, students were asked to determine an expression for $t_n$, as well as to find the value of $t_{10}$ and the values of $c$ and $d$ in the expressions $\log_c(t_n)$ and $\log_d(t_n)$. Students were provided with the general geometric sequence of $n$ terms and another sequence that had been formed by finding the logarithm of each term of the given geometric sequence. They were asked to write a logical argument to prove that this sequence of logarithmic expressions was arithmetic. An example of a common flaw in students' solutions included using specific values for the terms in writing a logical argument to show that $\log(a)$, $\log(ar)$, $\ldots\log(ar^{n-1})$ is arithmetic with a common difference of $\log r$.

Of the students who achieved the acceptable standard on the examination, 78% scored 3 or more marks out of 5, and of the students who achieved the standard of excellence on the examination, 78% scored 4 or more marks out of 5.

On this 5-mark question, the average mark was 2.7 or 54%.

## *Scoring Guide for Written-Response Questions*

Credit may be given to students who show unusual insight. If their solutions fall outside *Specific Question Scoring Rubrics*, they were scored against the *General Scoring Guide*.

### SPECIFIC QUESTION SCORING RUBRICS

### Question 1

**5** The student
- identifies the type of graph paper that would be used to sketch Case 1 and Case 2 and clearly outlines the steps required to sketch an ellipse for either Case 1 or 2. The steps are clearly communicated.

**4** The student
- identifies the type of graph paper for Case 1 and 2 and outlines the steps for one case. There is a minor flaw in the student's work

**or**
- identifies the type of graph paper for one case and outlines the steps for one case.

**3** The student
- identifies the type of graph paper for one case and outlines the steps required to sketch an ellipse for this case with a flaw

**or**
- identifies the type of graph paper for Case 1 and 2 and correctly draws the graph for either Case 1 or 2 with little or no explanation

**or**
- identifies the type of graph paper for Case 1 and 2 and makes significant progress on an explanation for one case.

**2** The student
- identifies the type of graph paper for Case 1 and Case 2

**or**
- begins to outline the steps for one case, using the correct graph paper.

**1** The student explores the initial stages of the problem.

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## Question 2

**5** The student
- completes the chart correctly **and**
- evaluates $\sum_{k=1}^{12} f(k)$ **and**
- explains the meaning of ${}_4C_2 \times {}_2C_2$ using the context of the problem **and**
- determines and justifies $g(n) = \frac{2^n}{2}$ or $g(n) = 2^{n-1}$

**4** The student's solution demonstrates a good understanding of the problem by
- correctly completing 3 of the 4 bullets **or**
- completing bullets 1 and 2; however, there is a weak explanation of ${}_4C_2 \times {}_2C_2$ **and/or** a weak justification in bullet 4

**3** The student demonstrates some understanding and finds partial solutions by
- completing the chart **and** one other bullet **or**
- completing the chart **and** making significant progress on at least 2 other bullets **or**
- explaining the meaning of ${}_4C_2 \times {}_2C_2$ within the context of the problem **and** demonstrating some understanding on another bullet **or**
- completing the 4th bullet **and** showing some understanding on another bullet

**2** The student explores initial stages and applies some relevant mathematical knowledge by
- completing the chart correctly **and** demonstrating some understanding on another bullet **or**
- correctly completing one of bullets 2, 3, or 4

**1** The student explores the initial stages of the problem.

## Question 3

**5** The student
- determines an expression for $t_n$ and the value of $t_{10}$ **and**
- determines correct values for $c$ and $d$ **and**
- completes a logical argument to show that $\log(a), \log(ar), \ldots \log(ar^{n-1})$ is arithmetic with a common difference of $\log r$

**4** The student demonstrates a good understanding of the problem by
- determining $t_{10}$, $t_n$, $c$, and $d$ **and** a start is made on the argument **or**
- completing the argument, using the general case, but only gets 3 of the 4 ($t_{10}$, $t_n$, $c$, and $d$)

**3** The student demonstrates some understanding by
- determining 3 or 4 of $t_{10}$, $t_n$, $c$, and $d$ **or**
- determining a correct value for 2 of the 4 ($t_{10}$, $t_n$, $c$, and $d$) and beginning the proof (may use specific values for the terms) **or**
- providing a logical argument to show $\log(a), \log(ar), \ldots \log(ar^{n-1})$ is arithmetic, stating $d = \log r$ (must be argued in general terms)

**2** The student
- determines the value of $t_{10}$ and an expression for $t_n$ **or**
- determines the value of $c$ and/or $d$ **or**
- provides an incomplete argument for the fourth bullet

**1** The student explores the initial stages of the problem

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For further information, contact Kathy McCabe (kmccabe@edc.gov.ab.ca) or Corinne McCabe (cmccabe@edc.gov.ab.ca) at the Student Evaluation Branch at 427-0010. To call toll-free from outside of Edmonton, dial 310-0000.

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